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Abstract

A systems theory has been developed (Part I) to interpret droplet size distributions in turbulent clouds by utilizing ideas from statistical physics and information theory. The present paper generalizes the systems theory to allow for varying fluctuations. The generalized theory provides a self-consistent theoretical framework for a wide range of fluctuations. It reduces to that presented in Part I when liquid water content is conserved, and becomes consistent with the uniform growth models for non-turbulent, adiabatic clouds. The theory indicates that there exists an important characteristic scale, defined as *saturation scale*, beyond which droplet size distributions do not change with further increasimpin averaging scale, but below which droplet size distributions strongly depend on the scale over which they are sampled and are therefore _ill-

1. Introduction

Reliable knowledge of cloud droplet size distributions is crucial for many cloudsome spectral broadening. Despite their differences, all these models have one feature in common: they attempt to follow each droplet or each parcel and then take statistical averages one way or another. By analogy to the kinetic theory of gases, these models are referred to as kinetic theories of droplet size distributions in the rest of the paper. Although these kinetic models produce size distributions broader than those predicted by uniform models and improve the understanding of the formation of droplet size distributions, the details of the processes involved are poorly understood and highly controversial.

It has been generally accepted that the fundamental equations describing individual droplets have been well established. However, to numerically solve these

theory approach. As will be shown, the newly generalized systems theory establishes a self-consistent theoretical framework, and offers new insights into the issues of spectral broadening and scale-dependence of droplet size d TDe ory establishes a with observed or modeled droplet size distributions in many cases. Therefore, a complete establish a generalized systems frameworQ applicable to a wide range of fluctuations by

$$H = -\int r(x)\ln(r(x))dx \tag{3}$$

The MXSD is the droplet size distribution that maximizes (3) subject to the constraints described by (1a) and (1b). By solving the corresponding variational problem, the MXSD was derived to be the Weibull distribution

$$n_{max}(D) = N_0 D^{b-1}\exp(-l D^b), \tag{4}$$

where the parameters $N_0 = ab/\beta$ and $\lambda = a/\beta$, and $\beta = X/N$. Note that the β here is the inverse of that used in Liu and Hallett (1997) and represents the mean value of X per droplet. This change makes the physical meaning of β consistent with that of "K_BT" in the Boltzmann energy distribution (K_B is the Boltzmann constant, T is the temperature, and K_BT essentially represents the mean energy per molecule in the gas). The W n ation of (4) also uses the power-law relationship (2).

The MNSD is associatin with the populational energy change (E) to form a population of droplets with n(D). In Part I, E is expressed as

$$E = -\frac{{}_{w}L}{6}\int D^3 n(D)dD + \int D^2 n(D)dD + c, \tag{5a}$$

where the first term on the right side is the latent energy with L representing the latent heat of water; the second term is the surface energy with σ representing the surface tension of 86.2. The coefficient c is related to the act ated CCN. Equation (5a) is W n ed u8W n the comma)assumption that othel forms of energy (i.e., gravitational potential energy, the kinetic energy associatin with droplet terminal velocities, and the solution effect) are negligibly small (Pruppachel and Klett 1978). In fact, these minor terms can be incorporatin into the coefficients before the integrals,

$$E = c_1 \int D^3 n(D) dD + c_2 \int D^2 n(D) dD + c \quad . \tag{5b}$$

The coefficient c

a. Scale-

physics, turbulence and related scale issues are poorly represented (if at all) in current models.

distribution is defi

techniques have been developed to measure droplet size distributealp as well as turbulent properties such as velocity variance and turbulent dissipateal rate (Babb and Verlinde

In the quest to understand and explain observe scale. Second, there exists a characteristic scale, defined as *saturation scale* of scale-

It is noteworthy that there may also be utility in applying the arguments presented in this paper for comparison of dynamical systems characterized by motions and particle distributions in an astronomical scale setting where fluctuations exist.

where $D_b = \left[\frac{\int D^b n(D)dD}{N}\right]^{1/b}$ is defined as the b-th diameter. The MNSD is the characteristic distribution

(A3) and (A4). Following the general procedure of variational calculus, we construct the specific Lagrangian functional as

$$F[n(D)] = c_1 \int D^3 n(D)dD + c_2 \int D^2 n(D)dD - l_1[\int n(D)dD] - l_2[\int D^b n(D)dD] \cdot$$

Setting the first variation of F[n(D)] with respect to the unknown n(D) equal to zero, we have

$$\Delta F = \int (c_1 D^3 + c_2 D^2 - l_1 - l_2 D^b) \Delta n(D)dD = 0 ,$$

Or, $l_1 \quad l_2 \quad \frac{}{dD} dD = 0 \cdot$ (A5)

Solving (A5) utilizes the knowledge of the generalized function as introduced above.

When the test function is chosen such that

$$\left\{ \begin{array}{l} 1, \; D < D_{min}, \ldots \\ \quad \ldots \; D_{min} \le \; \le D_{max} \end{array} \right.$$

d

$$\int n(D)\frac{d}{dD}dD = \int_{min}^{max}\left(3c_1D^2 + 2c_2D - \quad {}_2D \quad {}^1\right)n(D)dD = 0$$

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